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QPSK:

Receive Processing
for QPSK.

independent
Symbols.

$$x(t) = a_1 p_1(t) + a_2 p_2(t)$$

$$h_1(t) = p_1(T-\tau)$$

$$h_2(t) = p_2(T-\tau)$$

$$a_1 \epsilon_p + \tilde{n}_1$$

$$a_2 \epsilon_p + \tilde{n}_2$$

Matched Filter wrto $h(t) = P_1(T-\tau)$
AWGN channel.

$$y(t) = x(t) + n(t)$$

↑ Mean = 0
 $R_{nn}(\tau) = \frac{N_0}{2} \delta(\tau)$

$$y(t) = a_1 p_1(t) + a_2 p_2(t) + n(t)$$

↑ $h_1(t) = P_1(T-\tau)$

Matched Filter wrto this

$$\underbrace{y(t) * h(t)}_{r_1(t)} \Big|_{t=T} = \underbrace{x(t) * h_1(t)}_{\text{Signal}} \Big|_{t=T} + \underbrace{n(t) * h_1(t)}_{\text{Noise component}} \Big|_{t=T}$$

$$\begin{aligned} & (a_1 p_1(t) + a_2 p_2(t)) * h_1(t) \\ &= \int_{-\infty}^{\infty} (a_1 p_1(\tau) + a_2 p_2(\tau)) \cdot h_1(t - \tau) d\tau \\ &= \int_{-\infty}^{\infty} (a_1 p_1(\tau) + a_2 p_2(\tau)) \cdot p_1(t + \tau - T) d\tau \\ & \quad \begin{aligned} & h_1(t) = p_1(T - t) \\ & \Rightarrow h_1(t - \tau) = p_1(T + \tau - t) \end{aligned} \end{aligned}$$

✓ Sample at $t = T$

$$= \int_{-\infty}^{\infty} (a_1 p_1(\tau) + a_2 p_2(\tau)) p_1(\tau) d\tau$$

$$= a_1 \underbrace{\int_{-\infty}^{\infty} p_1^2(\tau) d\tau}_{E_p = 1} + a_2 \int_{-\infty}^{\infty} p_2(\tau) p_1(\tau) d\tau$$

inner product
= 0
Since $p_1(\tau), p_2(\tau)$
are orthogonal.

$$= a_1 E_p = a_1.$$

$$r_1(T) = a_1 + \underbrace{n(t) * h_1(t)}_{\tilde{n}_1} \Big|_{t=T}$$

$$r_1(T) = a_1 + \tilde{n}_1$$

$$a_1 = \pm A$$

$$A = \sqrt{E_b}$$

Gaussian
mean = 0
var = $\frac{N_0}{2} E_p$

= $\frac{N_0}{2}$
since $E_p = 1$

Is similar
to BPSK.
Optimal Detector = $r_1(T) \geq 0$

Decide $a_1 = A$
 if $r_1(T) \geq 0$.

Decide $a_1 = -A$
 if $r_1(T) < 0$.

Optimal Decision Rule.

Similarly, For a_2 .

$y(t) = \underbrace{x(t)}_{a_1 p_1(t) + a_2 p_2(t)} + n(t)$

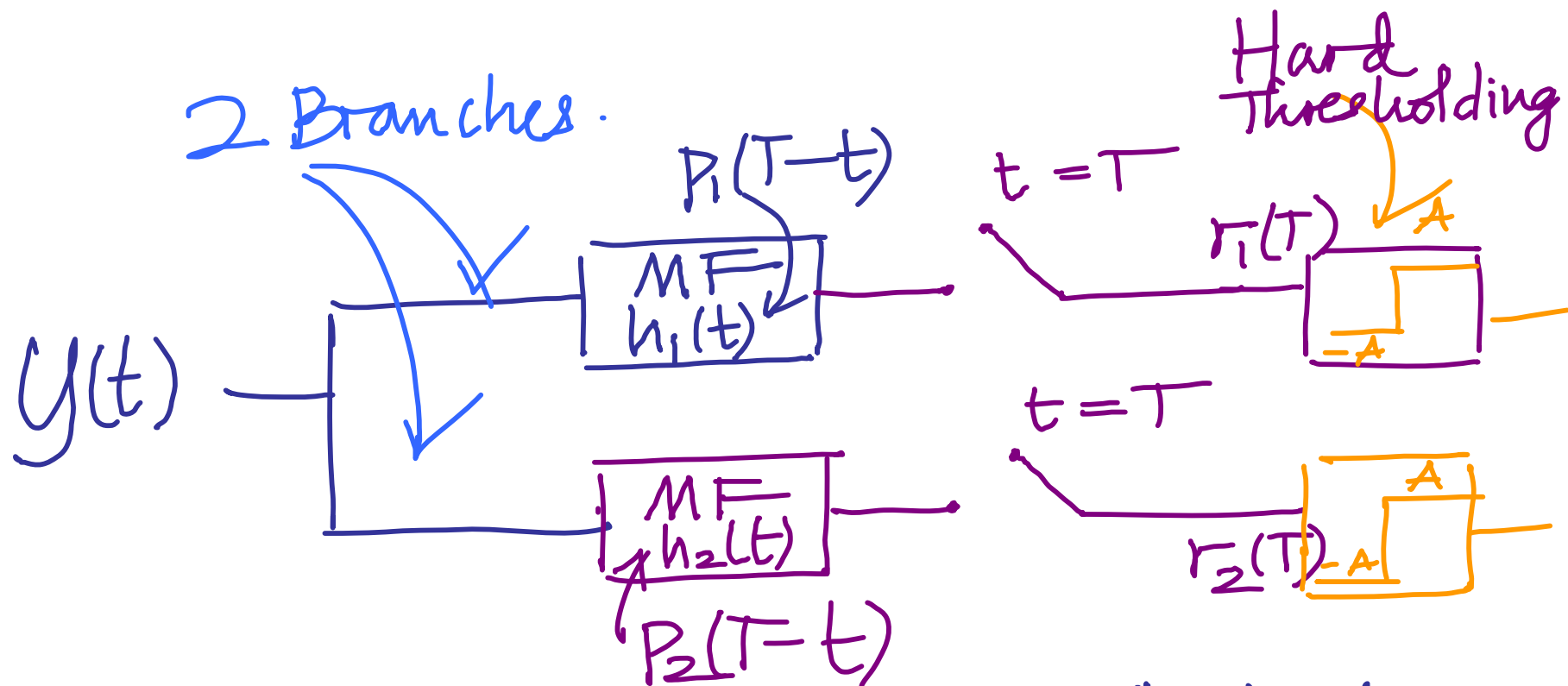
Matched Filter with $h_2(t) = p_2(T-t)$

$$\downarrow a_2 e_p + \tilde{n}_2$$

$$= a_2 + \tilde{n}_2$$

Gaussian
mean = 0
var = $\frac{N_0}{2}$

2 Branches.



Schematic For QPSK Detection

$$r_1(t) = a_1 + \tilde{n}_1$$

$\underbrace{\hspace{10em}}_{\pm A}$

$$r_2(t) = a_2 + \tilde{n}$$

$\underbrace{\hspace{10em}}_{\pm A}$

→ Bit Error rate of each channel is similar to BPSK.

BER of 2nd channel

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Overall symbol Error Rate

Probability that QPSK symbol is in error.

symbol is in error if either bit a_1 or a_2 is in error.

P_e of 1st channel = P_e
 P_c of 1st channel = $1 - P_e$.
Probability a_1 is received correctly

Similarly, probability of correct reception of a_2
= $1 - P_e$.

Probability that both bits a_1, a_2 are received correctly
= $(1 - P_e)(1 - P_e)$

$$= (1 - P_e)^2$$

They are independent
Events

$$P(A \cap B) = P(A) \cdot P(B)$$

a_1 is
received
correctly

a_2 is received
correctly

$$P_A = 1 - P_e$$

$$P_B = 1 - P_e$$

$$P(A \cap B) = (1 - P_e)^2$$

Probability symbol is in error
= $1 - \text{Prob both are received correctly}$

$$= 1 - (1 - P_e)^2$$

$$= 2P_e - P_e^2$$

$$P_e = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right) - Q^2\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Probability of overall Symbol Error

For high SNR, $\frac{E_b}{N_0}$

$$Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \ll 1$$

significantly
smaller than 1

$$\Rightarrow Q^2\left(\sqrt{\frac{2E_b}{N_0}}\right) \ll Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_e \approx 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

overall Prob
of Symbol
Error for
QPSK.